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A STUDY OF THE PROPAGATION OF ELASTIC WAVES IN WOUND STRUCTURES TAKING INTO ACCOUNT THEIR ROTATION UNDER EXTENSION†

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The propagation of weak- and strong-discontinuity longitudinal–torsional and transverse waves in various wound structures (helical ropes, flexible cables, textile yarns, and some other composite yarns) is considered. The tension and the torsional moment produced in cross-sections of wound structures (WS) by stretching are assumed to be functions of the relative deformation and torsional deformation. The wave propagation velocities, differential conditions, and discontinuities on the characteristic curves in a non-linearly WS are investigated. Problems of a constant-velocity longitudinal shock on a linearly elastic WS of semi-infinite and finite length are considered.

MOST wound structures (WS) rotate about their own axis when stretched. This basic property was accounted for by using a model [1–4] in which the tension and the torsional moment associated with stretching were assumed to be linear functions of the tensile and torsional deformation. In this paper, we assume that the tension and the torsional moment are non-linear functions of the tensile and torsional deformation.

1. MATHEMATICAL MODEL AND DIFFERENTIAL EQUATIONS OF THE PLANE MOTION OF A WOUND STRUCTURE (WS)

The main characteristic property of most WS is their ability to twist around their own axis when subjected to simple stretching. For instance, if an external tensile force is applied to the lower end of a vertically suspended right-twist textile yarn (TY), the yarn will rotate clockwise; a left-twist TY will rotate counterclockwise. If a force varying as shown in Fig. 1 is applied to the free end of the textile yarn, then for $0 \leq t \leq t^*$ the yarn will rotate with a positive angular acceleration and for $t > t^*$ it will roatate with a negative angular acceleration. The lower part of a freely suspended yarn acquires a helical shape in equilibrium under the action of its own weight. These and other experimental observations [3] suggest that the yarn has some initially unbalanced internal stresses,

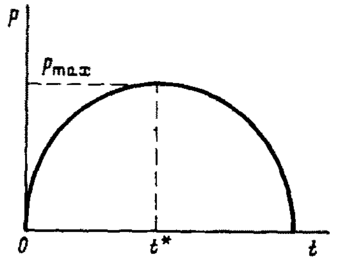

FIG. 1.

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and simple stretching produces simultaneously an axial force and a torsional moment in the yarn cross-sections. Similar effects are observed in most wound structures—helical ropes [2], commercial cables [4, 5], and some composites [6–8].

In the theory of the torsion of homogeneous rods, the displacements of points in a non-circular rod subjected to torsion are decomposed into two components [9]. First, the rod cross-sections rotate (like a rigid whole) about the axis of rotation; second, all the points are displaced along the axis of rotation. Arguing similarly, we can decompose the total relative strain of a WS into two components: the strain ε^* acquired by the WS due to stretching without rotation and the strain ε^{**} associated with rotation, i.e. $\varepsilon = \varepsilon^* = \varepsilon^{**}$, where $\varepsilon^{**} > 0$ when the yarn is untwisting, $\varepsilon^{**} < 0$ when the yarn is twisting, and $\varepsilon^{**} = 0$ for an ideal (neutral) yarn. The rotation about its own axis produced by stretching has been allowed for by setting $\varepsilon^{**} = A\theta$ [1, 2, 5, 6, 10, 14], which leads to the following model of a linearly elastic WS:

$$T = A_{11}\varepsilon + A_{12}\theta, \quad M = A_{21}\varepsilon + A_{22}\theta \quad (\theta = \partial\psi/\partial s) \tag{1.1}$$

where A is a coefficient of proportionality, T is the tension, θ is the torsional deformation, ψ is the angle of rotation of the cross-sections, M is the torsional moment, A_{ij} are the elastic coefficients (i, $j = 1, 2$) and s is the Lagrangian coordinate (in the model of a flexible cable [5] the torsional deformation θ is replaced with shearing deformation; the equation of compatibility can be used to transfer to θ). The coefficients A_{ij} allow for the physical properties of the material and the technological parameters of the WS. Methods of determining these coefficients are known for naturally twisted yarn [1], helical ropes [2], and flexible cables [5].

Using the model (1.1), we can model a non-linearly elastic WS in the form

$$T = T(\varepsilon, \theta), \quad M = M(\varepsilon, \theta) \tag{1.2}$$

i.e. we assume that the tension and the torsional moment in an arbitrary cross-section are non-linear functions of the total relative deformation and torsional deformation.

Assume that the stress in an arbitrary cross-section is directed along the tangent to the torsion axis, the principal moment in each section lies in the plane perpendicular to the torsion axis, and the moment of inertia of the cross-section relative to the torsion axis is constant for all points in the process of motion. We will use the following notation for arbitrary unknown functions:

$$F^{\cdot\cdot} = \frac{\partial^2 F}{\partial t^2}, \quad F'' = \frac{\partial^2 F}{\partial s^2}, \quad F_\alpha = \frac{\partial F}{\partial \alpha}, \quad d_\alpha F = \frac{dF}{d\alpha}$$

The equilibrium conditions of a WS element under the action of dynamic loads are described by the law of conservation of momentum expressed in projections on the Cartesian axes x and y,

$$\rho x^{\cdot\cdot} = (T\cos\varphi)', \quad \rho y^{\cdot\cdot} = (T\sin\varphi)' \tag{1.3}$$

and by the law of conservation of angular momentum expressed relative to the torsion axis

$$I\psi^{\cdot\cdot} = M' \tag{1.4}$$

where ρ is the initial density, $\varphi(s, t)$ is the angle between the tangent to the WS axis and the x-axis and I is the reduced moment of inertia of the cross-section relative to the torsion axis. The total relative strain satisfies the following geometrical equations [10, 11]:

$$(1 + \varepsilon)\cos\varphi = 1 + x', \quad (1 + \varepsilon)\sin\varphi = y' \tag{1.5}$$

Equations (1.3)–(1.5), jointly with Eq. (1.1) or (1.2), form a system of equations for the unknowns T, M, x, y, ε, θ, φ.

2. THE CHARACTERISTICS OF SYSTEM (1.2)–(1.5) AND THE DIFFERENTIAL CONDITIONS ON THE CHARACTERISTICS

System (1.2)–(1.5) can be written in the form

$$\begin{aligned} \rho x^{\cdot\cdot} &= \alpha_1 x'' + \beta_1\psi'' + \gamma_1 y'' \\ \rho y^{\cdot\cdot} &= \alpha_2 x'' + \beta_2\psi'' + \gamma_2 y'' \\ I\psi^{\cdot\cdot} &= \alpha_3 x'' + \beta_3\psi'' + \gamma_3 y'' \end{aligned} \tag{2.1}$$

$$\alpha_1 = \alpha^2 T_\varepsilon + \rho\beta b^2, \quad \beta_1 = \alpha T_\theta, \quad \gamma_1 = \alpha_2 = \alpha\beta\,(T_\varepsilon - \rho b^2),$$
$$\beta_2 = \beta T_\theta, \quad \gamma_2 = \beta^2 T_\varepsilon + \rho\alpha^2 b^2, \quad \alpha_3 = \alpha M_\varepsilon, \quad \beta_3 = M_\theta$$
$$\gamma_3 = \beta M_\varepsilon, \quad \alpha = \frac{1+x'}{1+\varepsilon}, \quad \beta = \frac{y'}{1+\varepsilon}, \quad b^2 = \frac{T}{\rho(1+\varepsilon)}$$

The characteristic determinant of system (2.1) leads to an equation, which has the following solutions:

$$k^2 = b^2 = T^*\,(1+\varepsilon)^{-1} \tag{2.2}$$

$$k^2 = a_{1,2}^2 = {}^1/_2\,(M_\theta^* + T_\varepsilon^* \pm \eta) \tag{2.3}$$

$$\eta = [(T_\varepsilon^* - M_\theta^*)^2 + 4T_\theta^* M_\varepsilon^*]^{1/2} \tag{2.4}$$

$$k^2 = (d_t s)^2, \quad T^* = T\rho^{-1}, \quad M^* = MI^{-1}$$

The dynamic load propagates along the WS in the direction of increasing values of the parameter s and in the opposite direction with three different velocities $\pm b$, $\pm a_1$, $\pm a_2$. System (2.1) is obviously hyperbolic if $a_1^2 \geqslant 0$ and $a_2^2 \geqslant 0$. It follows from relationships (2.3) and (2.4) that these conditions are satisfied if

$$T_\theta M_\varepsilon - T_\varepsilon M_\theta \geqslant 0 \tag{2.5}$$

We will assume that condition (2.5) holds at any instant.

The conditions on the characteristic curves are obtained by the following artificial technique. Assume that in addition to (2.1) we have the equations

$$\begin{aligned} dx^{\cdot} - k\,dx' &= \varphi_1(s,t)\,dt, \quad dy^{\cdot} - k\,dy' = \varphi_2(s,t)\,dt \\ d\psi^{\cdot} - k\,d\psi' &= \varphi_3(s,t)\,dt \end{aligned} \tag{2.6}$$

where φ_i are as yet unknown functions ($i = 1, 2, 3$). The functions φ_i in general are not all zero at the same time, since otherwise Eqs (2.1) could be represented as independent homogeneous wave equations describing, respectively, pure longitudinal, transverse, and torsional motion of the WS. Eliminating the derivatives with respect to t, we can rewrite (2.1) and (2.6) in the form

$$\begin{aligned} \rho\varphi_1 &= (\alpha_1 - \rho k^2)\,x'' + \beta_1\psi'' + \gamma_1 y'' \\ \rho\varphi_2 &= \alpha_2 x'' + \beta_2\psi'' + (\gamma_2 - \rho k^2)\,y'' \\ I\varphi_3 &= \alpha_3 x'' + (\beta_3 - Ik^2)\,\psi'' + \gamma_3 y'' \end{aligned}$$

Multiply the second and the third equations of the last system by the unknown coefficients λ^* and μ^*, respectively, and add the resulting equations. We now stipulate that the coefficients of the derivatives x'' and ψ'' are zero (because λ^* and μ^* are arbitrary coefficients). As a result, we obtain the equations

$$\begin{aligned} &I\,(\alpha_1 - \rho k^2) + \lambda^* I\alpha_2 + \mu^*\rho\alpha_3 = 0 \\ &I\beta_1 + I\lambda^*\beta_2 + \mu^*\rho\,(\beta_3 - Ik^2) = 0 \\ &\varphi_1 + \lambda^*\varphi_2 + \mu^*\varphi_3 = \left[\frac{\gamma_1}{\rho} + \lambda^*\frac{\gamma_2 - \rho k^2}{\rho} + \mu^*\frac{\gamma_3}{I}\right] y'' \end{aligned} \tag{2.7}$$

The left-hand side of the last equation in (2.7) and the coefficient of y'' are zero, because the second derivatives of the required functions have infinitely many values on the characteristic curves. Therefore, the following differential conditions hold on the characteristic curves (2.2)–(2.4):

$$dx^{\cdot} = k\,dx' - \lambda^*\,(dy^{\cdot} - k\,dy') + (k\,d\psi' - d\psi^{\cdot})\,\mu^* \tag{2.8}$$

From relationships (2.7) it follows that λ^* and μ^* are functions of k^2. Therefore, the three values of k^2 at each instant correspond to three values of λ^* and μ^* and there are a total of six differential conditions on the wave fronts $\pm b$, $\pm a_1$, $\pm a_2$.

Consider the linearly elastic case. With plane motion of the WS we obtain on the front $\pm a_{1,2}$

$$\lambda_{1,2}^* = \operatorname{tg} \varphi$$

$$\mu_{1,2}^* = \frac{IA_{12}(a_{1,2}^2 - b^2)}{\alpha\left[A_{21}A_{12} - (A_{11} - \rho b^2)(A_{22} - Ia_{1,2}^2)\right]}$$

and on the front $\pm b$

$$\lambda_3^* = \operatorname{ctg} \varphi, \quad \mu_3^* = 0$$

If the coefficients A_{ij} $(i, j = 1, 2)$ and the angle φ are constant, then introducing the new functions

$$w_i = x + \lambda_i^* y + \mu_i^* \psi \quad (i = 1, 2, 3) \tag{2.9}$$

we replace system (2.1) by the following homogeneous wave equations:

$$\ddot{w_i} = k_i^2 w_i'' \tag{2.10}$$

For rectilinear motion of the WS, we have $\varphi = 0$, $y = 0$, $\lambda_i^* = 0$, $\beta = 0$, $\alpha = 0$, and the functions w_i take the form

$$w_i = x + \mu_i^* \psi \quad (i = 1, 2) \tag{2.11}$$

From Eqs (2.9)–(2.11) it follows that the waves $\pm a_{1,2}$ are longitudinal–torsional and the waves $\pm b$ are transverse. The torsional strain affects the velocity of propagation of the transverse wave only through the tension [see (2.3)], and the higher the torsional deformation, the higher is the velocity of the transverse wave. Therefore, large torsional deformations (high velocities of the transverse wave) correspond to large relative deformations.

We can use Eq. (2.10) to show [10–12, 15] that when stationary loads are instantaneously applied to a linearly elastic WS, the excited regions in the WS are constant-parameter regions of rectilinear shape.

3. DISCONTINUITIES ON THE CHARACTERISTICS

It has been shown [11–14] that weak longitudinal discontinuities affect the tangential accelerations and the rate of elongation of an ideal yarn, without altering the normal accelerations and the rotational velocity of the yarn; weak transverse discontinuities alter only the rotational velocity and the normal accelerations of the yarn. In our case, these fundamental conclusions are not obvious, because our model of a flexible WS and the resulting scheme of the wave process are considerably different from the model and the wave-motion scheme of an ideal yarn.

We will use the traditional method to analyse discontinuities on the characteristics (see [11, 15]). Assume that the first derivatives of the displacements are continuous on the characteristic curves. Denote by $\mathbf{q}(s,t)$ and $n(s,t)$ the discontinuity coefficients of the first and second derivatives, respectively. The discontinuity coefficients of the unknown functions are given corresponding subscripts. We write Eqs (2.1) for the points to the left and to the right of the discontinuity line:

$$\begin{aligned} (\rho k^2 - \alpha_1)\, n_x - \beta_1 n_\psi - \gamma_1 n_y &= 0 \\ \alpha_2 n_x + \beta_2 n_\psi - (\rho k^2 - \gamma_2)\, n_y &= 0 \\ \alpha_3 n_x + \gamma_3 n_y - (Ik^2 - \sigma_3)\, n_\psi &= 0 \end{aligned} \tag{3.1}$$

The determinant of this system is zero. As the independent equations, we take the first and third equations in system (3.1).

Consider the transverse wave front. Let $k^2 = b^2$, $k^2 \neq a_{1,2}^2$. Multiply the first equation in (3.1) by M_ε and the second by $\alpha(T_\varepsilon - \rho b^2)$ and take the sum of these equations

$$[T_\theta - (Ib^2 - M_\theta)(\rho b^2 - T_\varepsilon)]\, n_\psi = 0 \tag{3.2}$$

The multiplier of n_ψ in Eq. (3.2) does not vanish by virtue of Eq. (2.2) and the assumption $k^2 = b^2$. Substituting $n_\psi = 0$ in the first and third equations in (2.1), we obtain

$$n_x = -n_y \operatorname{tg} \varphi, \ [\theta'] = [\psi''] = 0, \ [x''] = -\operatorname{tg} \varphi\, [y''] \tag{3.3}$$

Now multiply the first equation in (1.4) by $\cos\varphi$ and the second by $\sin\varphi$ and take the sum of these equations

$$q_\varepsilon = (n_x \cos \varphi + n_y \sin \varphi)\, v' \tag{3.4}$$

where $v(s, t) = 0$ is the equation of the characteristic curve.

Assume that $v' \neq 0$ (stationary discontinuity fronts are not considered). The right-hand side of Eq. (3.4) is zero by the first equality in (3.3). Therefore, on the front $\pm b$ we have $q_\varepsilon = 0$.

Now multiply the first equation in (1.5) by $\sin\varphi$ and the second by $\cos\varphi$. Proceeding as before, we have

$$(1 + \varepsilon)\, q_\varphi = (n_y \cos \varphi - n_x \sin \varphi)\, v' \tag{3.5}$$

From the first equation in (3.3) and relationship (3.5), we obtain

$$q_\varphi = n_y v' \, [(1 + \varepsilon) \cos \varphi]^{-1} \tag{3.6}$$

Analysis of Eqs (3.3)–(3.6) shows that weak-discontinuity transverse waves do not affect the tensile and torsional deformations of the WS.

Let us now consider the discontinuity fronts $a_{1,2}^2$ propagating along an initially straight WS. Substituting into Eqs (1.5) and (3.1) $\beta = 0$, $\alpha = 0$, $k^2 = a_{1,2}^2$, we obtain

$$\begin{gathered} n_y = 0, \ n_x = n_\psi T_\theta\, (\rho a_{1,2}^2 - T_\varepsilon)^{-1} \\ q_\varphi = 0, \ q_\theta \neq 0, \ n_x = q_\varepsilon v' \end{gathered} \tag{3.7}$$

Weak discontinuities $\pm a_{1,2}$ obviously affect the tensile and torsional deformations, and also the accelerations of the points of the WS tangential to the elastic axis.

4. SPECIAL CASES

Case 1. Longitudinal waves in an ideal yarn propagate with velocity $d_\varepsilon T^*$. In our case, if we assume that $a_{1,2}^2 = d_\varepsilon T^*$, we obtain the following relationship between the total differentials of the tensile and torsional deformations:

$$d\theta = {}^1/_2\, (M_\theta^* - T_\varepsilon^* \pm \eta)\, (T_\theta^*)^{-1} d\varepsilon \tag{4.1}$$

The integrals of Eq. (4.1) corresponding to the positive and negative sign before the radical can be represented in the form

$$f_i\, (\varepsilon, \theta, c_i) = 0 \quad (i = 1, 2) \tag{4.2}$$

where c_1 and c_2 are integration constants. From (4.1) and (4.2) it follows that in this case there are two families of curves of opposite sign relative to the $O\varepsilon$ axis in the (ε, θ) plane. Substituting θ from Eq. (4.2) into $T = T(\varepsilon, \theta)$, we obtain two families of curves in the (T, ε) corresponding to the (θ, ε) curves. At each point, the slope that the tangent to the curve of the corresponding family makes with the $O\varepsilon$ axis is equal to the square of the velocity of propagation of the corresponding discontinuity a_1^2 or a_2^2. The slope to the $O\varepsilon$ axis of the line passing through a given point in the (T, ε) plane and the point $\varepsilon = -1$ on the $O\varepsilon$ axis equals the square of the velocity of propagation of the transverse wave [11, 12]. The sign of the right-hand side of Eq. (4.1) matches the sign before the radical. Therefore, if $T_\theta > 0$, then the deformation differentials have equal signs on the discontinuity front a_1^2 and opposite signs on the front a_2^2.

Case 2. Let

$$a_{1,2}^2 = d_\theta M^* \tag{4.3}$$

Considering Eqs (2.4) and (4.3) simultaneously, we find

$$d\varepsilon = {}^1/_2\, (T_\varepsilon^* - M_\theta^* \pm \eta)\, (M_\varepsilon^*)^{-1} d\theta \tag{4.4}$$

The integrals of this equation can be represented in the form

$$f_j\, (\varepsilon, \theta, c_j) = 0 \quad (j = 3, 4) \tag{4.5}$$

We see that if each equation in (4.5) is solved for ε and the solutions are substituted into the equation $M = M(\varepsilon, \theta)$, a similar wave pattern for $a_{1,2}^2$ is obtained in the (ε, θ) and (M, θ) planes.

Case 3. The most interesting case is

$$d_\varepsilon T^* = \varkappa, \; d_\theta M^* = \varkappa \tag{4.6}$$

Expanding the total differentials and equating the left-hand sides of Eq. (4.6), we obtain

$$T_\varepsilon^* + T_\theta^* d_\varepsilon \theta = M_\varepsilon d_\theta \varepsilon + M_\theta^* \tag{4.7}$$

Solving Eq. (4.7) for $d_\varepsilon \theta$ and $d_\theta \varepsilon$, we obtain Eqs (4.1) and (4.4), respectively. Substituting $d_\varepsilon \theta$ into the first expression in (4.6) and $d_\theta \varepsilon$ into the second one, we obtain $\varkappa = a_{1,2}^2$. Thus, from (4.6) we obtain

$$a_{1,2}^2 = d_\varepsilon T^* = d_\theta M^* \tag{4.8}$$

and cases 1 and 2 are special cases that follow from assumption (4.6). From Eqs (4.1), (4.2), (4.4) and (4.5), we see that in this case there are two families of curves in the (T, M) plane corresponding to the (ε, θ) curves.

Equalities (4.8) imply that the slopes of the tangents at the corresponding point (T^0, ε^0) and (M^0, θ^0) of the (T, ε) and (M, θ) planes are equal.

In other words, the longitudinal and torsional waves propagate with equal velocity along the WS, and the particles in the regions excited by the waves $\pm a_1$ and $\pm a_2$ execute complicated longitudinal–torsional motion.

5. PROPAGATION OF STRONG DISCONTINUITIES. A LONGITUDINAL SHOCK ON A LINEARLY ELASTIC WOUND STRUCTURE

Assume that for $t \geqslant 0$ a strong discontinuity wave arrives at the end of a WS element with initial length ds_0; the wave propagates from left to right with velocity D. During the time dt, this element changes its initial length $ds_0 = (1 + \varepsilon_0)dt$ to a new length $ds_1 = (1 + \varepsilon_1)dt$.

From the condition of continuity of the displacement vector $\mathbf{l}(s, t)$ and the angle of rotation of the cross-sections along the line $ds = Ddt$, we obtain

$$[\mathbf{l}^\cdot] + D\, [\mathbf{l}'] = 0, \; [\psi^\cdot] + D\, [\psi'] = 0 \tag{5.1}$$

These equations can be written in the form

$$\begin{aligned} \mathbf{l}_1^\cdot - \mathbf{l}_0^\cdot &= D\, (1 + \varepsilon_0)\, \boldsymbol{\tau}_0 - (1 + \varepsilon_1)\, \boldsymbol{\tau}_1) \\ \psi_1^\cdot - \psi_0^\cdot &= D\, (\theta_0 - \theta_1) \end{aligned} \tag{5.2}$$

Applying the laws of conservation of momentum and angular momentum to the element ds_0, we obtain

$$\begin{aligned} T_0^* \boldsymbol{\tau}_0 - T_1^* \boldsymbol{\tau}_1 &= D\, (\mathbf{l}_1^\cdot - \mathbf{l}_0^\cdot), \\ M_0^* - M_1^* &= D\, (\psi_1^\cdot - \psi_0^\cdot) \end{aligned} \tag{5.3}$$

Substituting (5.2) into the second equation in (5.3), we obtain

$$[T_0^* - D^2\, (1 + \varepsilon_0)]\, \boldsymbol{\tau}_0 = [T_1^* - D^2\, (1 + \varepsilon_1)]\, \boldsymbol{\tau}_1 \tag{5.4}$$

Consider the transverse wave front. Let $\tau_0 \neq \tau_1$. From Eq. (5.4) we obtain

$$T_0^* - D^2\, (1 + \varepsilon_0) = 0, \; T_1 - D^2\, (1 + \varepsilon_1) = 0 \tag{5.5}$$

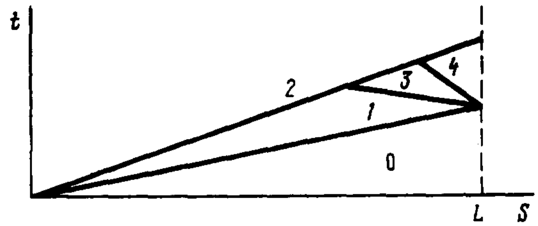

FIG. 2.

The system consisting of the second equations in (5.2), (5.3) and also (5.4), (5.5) has the following solution:

$$\varepsilon_1 = \varepsilon_0,\ T_1^* = T_0^*,\ \theta_1 = \theta_0,\ M_1^* = M_0^*$$
$$D^2 = T_0^* (1 + \varepsilon_0)^{-1},\ l_1^{\cdot} - l_0^{\cdot} = D (\tau_0 - \tau_1) \tag{5.6}$$

We see that strong- and weak-discontinuity transverse waves propagate with equal velocity along the WS and do not cause a deformation discontinuity.

Proceeding similarly for $\tau_0 = \tau_1$, we obtain from Eqs (5.2)–(5.6) the conditions

$$l_1^{\cdot} - l_0^{\cdot} = \mp D (\varepsilon_1 - \varepsilon_0) \tau_0,\ \psi_1^{\cdot} - \psi_0^{\cdot} = \mp D (\theta_1 - \theta_0) \tag{5.7}$$

on the longitudinal–torsional discontinuity fronts.

Assume that the left end of a linearly elastic and initially straight WS starts moving at $t \geqslant 0$ according to a given law

$$x^{\cdot} (0, t) = x_{00}^{\cdot} (t),\ \psi^{\cdot} (0, t) = \psi_{00}^{\cdot} (t)$$

Two longitudinal–torsional waves propagating with constant velocities a_1 and a_2 are generated in the WS. If $x_{00}^{\cdot}$ and $\psi_{00}^{\cdot}$ are constant (a constant-velocity shock), the perturbed regions 1 and 2 (Fig. 2) of the structure are constant-parameter regions [10–12]. On the wave fronts a_1 and a_2, we have the equations ($i = 1, 2$)

$$x_i^{\cdot} - x_{i-1}^{\cdot} = a_i (\varepsilon_{i-1} - \varepsilon_i),\ \psi_i^{\cdot} - \psi_{i-1}^{\cdot} = a_i (\theta_{i-1} - \theta_i) \tag{5.8}$$

$$\theta_i - \theta_{i-1} = \mu_i (\varepsilon_i - \varepsilon_{i-1}) \tag{5.9}$$

$$\mu_i = \left\{\frac{A_{22}}{I} - \frac{A_{11}}{\rho} \pm \left[\left(\frac{A_{11}}{\rho} - \frac{A_{22}}{I}\right)^2 + \frac{4}{\rho I} A_{11} A_{22}\right]^{1/2}\right\} \left(\frac{2A_{12}}{\rho}\right)^{-1}$$

Equations (5.9) follow from (4.1). System (5.8), (5.9) has the following solution:

$$\begin{gathered}
x_2^{\cdot} = x_{00}^{\cdot},\ \psi_2^{\cdot} = \psi_{00}^{\cdot} \\
\varepsilon_1 = \varepsilon_0 + (\mu_2 x_*^{\cdot} - \psi_*^{\cdot}) (a_1 \mu)^{-1} \\
\varepsilon_2 = \varepsilon_0 + (a^* x_*^{\cdot} - a \psi_*^{\cdot}) (a_1 a_2 \mu)^{-1} \\
\theta_1 = \theta_0 + (\mu_1 \mu_2 x_*^{\cdot} - \mu_1 \psi_*^{\cdot}) (a_1 \mu)^{-1} \\
\theta_2 = \theta_0 + (\mu_1 \mu_2 a x_*^{\cdot} - a^* \psi_*^{\cdot}) (a_1 a_2 \mu)^{-1} \\
x_1^{\cdot} = x_0^{\cdot} + (\mu_2 x_*^{\cdot} - \psi_*^{\cdot}) \mu^{-1} \\
\theta_1 + \theta_0 + (\mu_1 \psi_*^{\cdot} - \mu_1 \mu_2 x_*^{\cdot}) \mu^{-1} \\
\mu = \mu_1 - \mu_2,\ a = a_2 - a_1, \\
\psi_*^{\cdot} = \psi_{00}^{\cdot} - \psi_0^{\cdot},\ x_*^{\cdot} = x_{00}^{\cdot} - x_0^{\cdot} \\
a^* = a_2 \mu_1 - a_1 \mu_2
\end{gathered} \tag{5.10}$$

Assume that the WS is of finite length L and for $t = L/a_1$ the longitudinal–torsional wave a_1 is reflected from the rigidly clamped end at the point $s = L$. Also assume that the reflection of the wave a_1 produces two reflected longitudinal–torsional waves that propagate with the velocities a_1 and a_2 (Fig. 2). As before, we obtain

$$\begin{aligned}\varepsilon_3 &= 2\varepsilon_1 - \varepsilon_0 - (\psi_0^{\cdot} - \mu_2\chi_0^{\cdot})(a_1\mu)^{-1}\\ \varepsilon_4 &= \varepsilon_3 + (\psi_0^{\cdot} - \mu_1 x_0^{\cdot})(a_2\mu)^{-1}\\ \theta_3 &= 2\theta_1 - \theta_0 - \mu_1(\psi_0^{\cdot} - \mu_2 x_0^{\cdot})(a_1\mu)^{-1}\\ \theta_4 &= \theta_3 + \mu_2(\psi_0^{\cdot} - \mu_1 x_0^{\cdot})(a_2\mu)^{-1}\\ \psi_3^{\cdot} &= \mu_2(\mu_1 x_0^{\cdot} - \psi_0^{\cdot})\mu^{-1}, \ \psi_4^{\cdot} = 0\\ x_3^{\cdot} &= (\psi_0^{\cdot} - \mu_1 x_0^{\cdot})\mu^{-1}, \ x_4^{\cdot} = 0\end{aligned} \tag{5.11}$$

We see that in general reflection of the wave a_1 from a rigidly clamped end of the flexible WS produces two longitudinal–torsional waves.

Consider the following special case. Let $\psi_0^{\cdot} = 0$, $x_0^{\cdot} = 0$, $\varepsilon_0 = 0$, $\theta_0 = 0$. From (5.11) we obtain the relationships

$$\varepsilon_4 = \varepsilon_3 = 2\varepsilon_1, \quad \theta_4 = \theta_3 = 2\theta_1$$
$$\psi_4^{\cdot} = \psi_3^{\cdot} = 0, \quad x_4^{\cdot} = x_3^{\cdot} = 0$$

from which it follows that reflection of an elastic longitudinal–torsional wave from a rigidly clamped end of an initially unstrained and static WS produces only one reflected wave. In the latter case, the tensile and torsional deformations on the reflected wave front (as for a flexible yarn) are equal to twice the deformation on the direct wave.

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